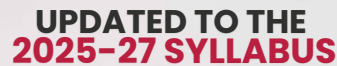




O Levels & IGCSE

FULL SYLLABUS NOTES

1st Edition

MOJZA

O levels & IGCSE

MATHEMATICS

NOTES

4024 & 0580



BY TEAM MOJZA

CONTENTS

Arithmetics ----- **Pg 02**

Algebra ----- **Pg 06**

Geometry ----- **Pg 09**

Mensuration ----- **Pg 11**

Matrices, Vectors & Transformation ----- **Pg 13**

Statistics ----- **Pg 15**

Probability ----- **Pg 17**

Arithmetics

Numbers Family:

- Natural numbers:

- All the positive integers from **1** to **Infinity**.
- Fractions or decimals are not included.
- They are also called Positive Integers
- E.g: **1, 2, 3, 4, 5, 6, ...**

- Whole Numbers:

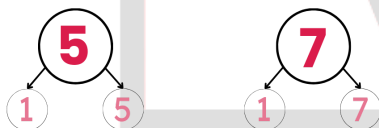
- All the positive Integers including **zero**
- E.g: **0, 1, 2, 3, 4, 5, 6, ...**

- Integers:

- Integers include Positive numbers, negative numbers or zero
- Most common word used by the examiner.
- **-3, -2, -1, 0, 1, 2, 3, ...**

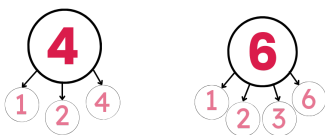
Prime Numbers:

- Numbers that have exactly 2 factors; **1** and the number **itself**.
- For Example:



- Composite Numbers

- Numbers that have more than two factors



- Rational numbers:

- Any whole number, that can be written as a **pure fraction** or **decimal**
- Numbers that can be expressed in the form of a ratio.
- They must either **end** (terminating decimals) or **repeat** (recurring decimals).
- For Example: $\frac{3}{4}$, **4**, **1.75** and **1.333333333**

- Irrational numbers:

- Any number that **cannot** be expressed in the form of a fraction.
- E.g: π , $\sqrt{2}$, $\sqrt{7}$

Exam-style Question

Write down an irrational numbers between 10 and 11.

Solution

10

↓

$\sqrt{100}$

Rational

and

11

↓

$\sqrt{121}$

Rational

$\sqrt{101}, \sqrt{102}, \dots, \sqrt{120}$

All of these are irrational

- Terminating Decimals

- Decimal numbers which end after a certain number of decimal places.
- E.g. $\frac{9}{8} = 1.125$ - ends after a certain decimal place, hence it is a terminating decimal.

- Recurring Decimals

- Decimal numbers which keep repeating a digit or group of digits
- For Example: $\frac{1}{3} = 0.3333333\ldots$
- $\frac{3}{11} = 0.272727\ldots$, is a recurring decimal. The two digits **27** repeat in this order.
- Recurring decimals are written with dots

- Real numbers:

- All the numbers that exist
- These include all rational and irrational numbers.

- Squared Numbers:

- Multiplying the number with itself
- The Square of the number is called a **Perfect Square**.
- Always **positive**.
- Candidates are required to memorise square numbers up till **15** for **Paper 1**.

$1^2 = 1 \times 1 = 1$	$11^2 = 11 \times 11 = 121$
$2^2 = 2 \times 2 = 4$	$12^2 = 12 \times 12 = 144$
$3^2 = 3 \times 3 = 9$	$13^2 = 13 \times 13 = 169$
$4^2 = 4 \times 4 = 16$	$14^2 = 14 \times 14 = 196$
$5^2 = 5 \times 5 = 25$	$15^2 = 15 \times 15 = 225$
$6^2 = 6 \times 6 = 36$	$16^2 = 16 \times 16 = 256$
$7^2 = 7 \times 7 = 49$	$17^2 = 17 \times 17 = 289$
$8^2 = 8 \times 8 = 64$	$18^2 = 18 \times 18 = 324$
$9^2 = 9 \times 9 = 81$	$19^2 = 19 \times 19 = 361$
$10^2 = 10 \times 10 = 100$	$20^2 = 20 \times 20 = 400$

- Cube numbers:

- Multiplying the number with itself that number of times.
- The cube of the number is called a **Perfect cube**.
- May be **positive** or **negative**.

$1^3 = 1 \times 1 \times 1 = 1$
$2^3 = 2 \times 2 \times 2 = 8$
$3^3 = 3 \times 3 \times 3 = 27$
$4^3 = 4 \times 4 \times 4 = 64$
$5^3 = 5 \times 5 \times 5 = 125$
$6^3 = 6 \times 6 \times 6 = 216$
$7^3 = 7 \times 7 \times 7 = 343$
$8^3 = 8 \times 8 \times 8 = 512$
$9^3 = 9 \times 9 \times 9 = 729$
$10^3 = 10 \times 10 \times 10 = 1000$

- Common factors:

- A **factor** that is shared between **two** different numbers.
- The HCF (**Highest** common factor) is the **largest** number that is a common factor of two different numbers.
- For Example, in the example below, the HCF of the two numbers is

$$24 = 2 \times 2 \times 2 \times 3$$

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

$$\text{HCF} = 2 \times 2 \times 2 \times 3 \quad \text{or} \quad 2 \times 2^2 \times 3$$

$$= 24$$

- Common multiples:

- A **multiple** that is common between **two** different numbers.
- A number that is present in the tables of two different numbers.
- LCM (**Lowest** common multiple) is the **smallest** number that is the multiple of two different numbers.
- In the example below, the **common multiples** of the two numbers is $5 \times 7 \times 9$, while the **LCM** of the numbers is **1575**.

$$175 = 5 \times 5 \times 7$$

$$225 = 5 \times 5 \times 9$$

$$\text{LCM} = 5 \times 5 \times 7 \times 9 \quad \text{or} \quad 5^2 \times 7$$

$$= 1575$$

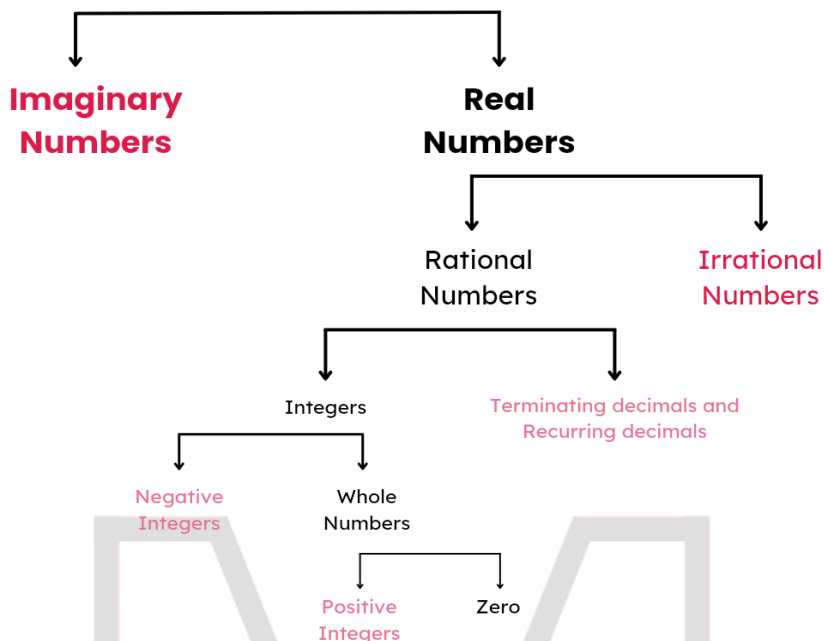
- Reciprocals:

- The number obtained by interchanging the numerators and denominators of the original number.

$$\left(\frac{9}{14} \right) \Rightarrow \frac{14}{9}$$

- When a number doesn't have a denominator, 1 is put to reciprocate it.

NUMBERS FAMILY



Sets:

- A set is an organised, unique **collection** of elements.
- You are **not** allowed to repeat the same element in a set.

- Examples of sets:

- **A** = {x: x is a natural number}
= {1, 2, 3, 4, ...}
- **B** = {x: x is an integer between 1 and 8}
= {2, 3, 4, 5, 6, 7}
- **C** = {x: $1 \leq x \leq 15$ }
= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}
- **D** = {a, b, c, ...}

- Set Notation

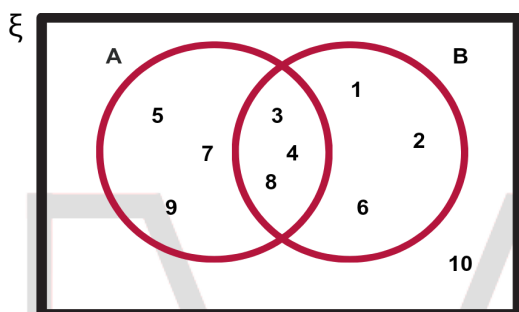
- ':' A colon should be read as '**Such that**'
- $\mathbb{U}$: Universal set
- $\in$: is an element of
- $\notin$: is not an element of
- **n(A)**: Number of elements in set A
- **A'**: Complement of set A (The set containing all the elements of the universal set **but** those of set A)
- $\emptyset$: The empty/null set
- **A $\cup$ B**: Union of A and B (The set produced by writing **all** the elements of both sets)

- **$A \cap B$** : Intersection of A and B (The set produced by noting down the elements **common** in both the sets)
- **$A \subseteq B$** : A is a subset of B
- **$A \not\subseteq B$** : A is **not** a subset of B
- **$A \subset B$** : A is a proper subset of B
- **$A \not\subset B$** : A is **not** a proper subset of B

- Venn Diagrams:

$$x : 1 \leq x \leq 15$$

$$\rightarrow \xi: \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$



- **A**: { 3, 4, 5, 7, 8, 9 }
- **B**: { 1, 2, 3, 4, 6, 8 }
- **A'**: { 1, 2, 6, 10 }
- **B'**: { 5, 7, 9, 10 }
- **$A \cup B$** : { 1, 2, 5, 6, 7, 9 }
- **$A \cap B$** : { 3, 4, 8 }
- **$A \cup B'$** : { 5, 7, 9, 10 }
- **$(A \cup B)'$** : { 10 }
- **$(A \cap B)'$** : { 1, 2, 5, 6, 7, 9, 10 }

Fractions, Decimals and Percentages:

Fractions:

- Proper Fractions:

- Fractions for which the numerator is smaller than the denominator.
- E.g: $\frac{2}{3}$, $\frac{1}{4}$, $\frac{5}{7}$

- Improper Fractions:

- Fractions for which the numerator is larger than the denominator.
- Mixed numbers can be converted into improper fractions in this way:

$$4\frac{1}{5} = \frac{21}{5}$$

$$1\frac{16}{23} = \frac{39}{23}$$

$$8\frac{7}{11} = \frac{95}{11}$$

→ E.g $\frac{4}{3}$, $\frac{8}{5}$

- Mixed Numbers:

- Mixed Numbers are simplified forms of improper fractions.
- Includes a whole number and a fraction.
- Improper fractions can be converted into mixed numbers by the following method:

$$\frac{17}{5} \rightarrow 5 \overline{) 17}$$

$$\begin{array}{r} 5 \overline{) 17} \\ - 15 \\ \hline 2 \end{array} \rightarrow 3\frac{2}{5}$$

- Decimals:

- Names for the decimal places always end with 'th', starting with 'tenth'.
- To convert a decimal number into a fraction:

$$\frac{0.7}{10} = \frac{7}{10}$$

- To convert a decimal number into a percentage, move the decimal place two places to the right.

$$\frac{0.32}{100} = \frac{32}{100} = \frac{8}{25}$$

$$\frac{0.125}{1000} = \frac{125}{1000} = \frac{1}{8}$$

- Percentages:

- A percentage is a **number** or **ratio** that can be expressed as a fraction of **100**.
- The **%** sign next to a number indicates that the number is being **divided** by **100**.
- E.g: $\frac{30}{100} = 30\%$

The Four Operations:

- BODMAS

- The BODMAS rule needs to be applied for solving any equation or simplifying any expression. It stands for:
 - **B**rackets
 - **O**perations
 - **D**ivision
 - **M**ultiplication
 - **A**ddition
 - **S**ubtraction
- The rule states that in any equation or expression:
 - The **first** step is to open up the **brackets** by solving it.
 - The second step is to simplify all the **operations**, such as the **powers** and **sin/cos/tan**
 - The **third** step is to do all the **division**.
 - The **fourth** step is the **multiplication**.
 - The fifth step is the addition.
 - The Sixth step is subtraction.

- Example:

$$\begin{aligned}
 &64 \div 8 \times 9 + 4^3 - (10 + 6) \\
 &= 64 \div 8 \times 9 + 4^3 - \underline{16} && \text{Brackets: } (10 + 6) \\
 &= 64 \div 8 \times 9 + \underline{64} - 16 && \text{Order of Powers: } 4^3 \\
 &= \underline{8} \times 9 + 64 - 16 && \text{Division: } 64 \div 8 \\
 &= \underline{72} + 64 - 16 && \text{Multiplication: } 8 \times 9 \\
 &= \underline{136} - 16 && \text{Addition: } 72 + 64 \\
 &= \underline{120} && \text{Subtraction: } 136 - 16
 \end{aligned}$$

- Positive and Negative signs:

- Two negative signs when present next to each other should be written as a positive sign.
- E.g $2--2 = 2+2 = 4$
- A positive and negative sign when present next to each other should be written as a negative sign.

- E.g $2+-2 = 2-2 = 0$
- When adding or subtracting numbers with opposite signs, the answer should have the sign of the larger number.
- E.g $-2+7 = +5$ (The positive sign does not need to be stated here)

- Note:

- Whether you divide or Multiply first, add or subtract first, the answer will remain the same.

Standard Form

- Numbers written in standard form are expressed in powers of 10. Standard form: The number $a \times 10^n$ is in standard form when $1 \leq a < 10$ and n is a positive or negative integer.
- The decimal must be shifted so that only **one** number is present to its **left**.
- The number of places the decimal jumps will be equal to the power of 10.
- Moving the decimal to the right will result in a negative power of 10.
- Moving the decimal to the left will result in a positive power of 10.
- Numbers in standard form are added and subtracted in the following manner:
- The powers of 10 must first be equalled.

Key:

Decimal Moves Right: Power Decreases

Decimal Moves left: Power Increases

0.000043



- To make the powers equal, either **subtract** or **add**.
- If you subtract, add the number to the power of 10
- $40 \times 10^4 - 5 \times 10^4$ or $4 \times 10^5 - 0.5 \times 10^5$
- The power of 10 must be taken common now.
E.g $(40 - 5) \times 10^4$
- Next, the bracket needs to be solved.
E.g 35×10^4
- Finally, the answer needs to be converted into standard form.
E.g 3.5×10^5
- Numbers in standard form are multiplied and divided in the following way:
- The powers of ten are either subtracted (If division is taking place) or added (If addition is taking place).
E.g $8 \times 10^5 \div 4 \times 10^2$
- $(8 \div 4) \times 10^3$
- The bracket is now solved.
E.g 2×10^3
- The answer should now be converted to standard form.
E.g 2×10^3 is already in standard form
- If the original problem includes numbers in decimals, they should first be converted to whole numbers.

E.g $4.8 \times 10^4 \div 4 \times 10^2$
 $48 \times 10^4 \div 4 \times 10^2$

Significant Figures:

- Rules:

- All **non-zero** digits are significant figures.
- Zeros **between** non-zero numbers are significant.
- Leading Zeros are **not** significant
- E.g 0.000043 only has 3 significant figures
- Trailing zeros may or may not be significant.
- E.g 1000 rounded to 2 s.f. is 1000
 1000 rounded to 3 s.f. is also 1000

- Rounding:

- Numbers may be rounded to decimal places, whole numbers or significant figures.
- Numbers may be rounded to tens, thousands, hundredths, tenths etc.

Limits of Accuracy:

- Upper bounds

- Data given is rounded off to a specific accuracy.
- To find the upper and lower bounds:
- The specified accuracy needs to be divided by 2.
- E.g 100m rounded to the nearest metre.
 $1 \text{ metre} / 2 = 0.5$
- The number obtained after dividing by 2 needs to be added (for the upper bound) and subtracted (for the lower bound) once.
- E.g Upper bound: $100 + 0.5 = 100.5$
 Lower bound: $100 - 0.5 = 99.5$

Number Sequence And Patterns

- Number Sequence

- Term: Each number in the sequence.
- First Term (a_1): The initial number.
- Common Difference (d): The constant difference between consecutive terms in an arithmetic sequence.
- nth Term Formula: $a_n = a_1 + (n-1)d$

2, 4, 6, 8, 10, ...

+2 +2 +2 +2

7, 14, 28, 56, 112, ...

$\times 2$ $\times 2$ $\times 2$ $\times 2$

0, 2, 5, 9, 14, 20, 27, ...

+2 +3 +4 +5 +6 +7 +8 +9

+1 +1 +1 +1 +1 +1 +1 +1

Ratio, Rate and Proportion:

- Ratio and Proportion:

- A ratio is a comparison of two quantities.
- Ratios are written as 2 : 4, 8 : 2
- They are simplified in the same way that fractions are.
- E.g $2 : 4 = 1 : 2$
- $1 : 2$ can also sometimes be written as $\frac{1}{2}$
- For a given ratio, adding all the numbers in the ratio will give you the total.
- E.g: A class has 4 boys and 10 girls
- Ratio of boys to girls = 4 : 10
- Total students = $4 + 10 = 14$
- Using this logic, we can find the number of boys or girls when a simplified ratio and the total number of students are given.

- Example:

- A class has 40 students
- The ratio of girls to boys in the class is 3 : 5
- To find the numbers of girls and boys, we will:
- Add the numbers in the ratio: $3 + 5 = 8$
- Divide the number of students by this number: $40/8 = 5$
- We will now multiply both the numbers in the ratio by this number.
- $3 \times 5 : 5 \times 5 = 15 : 25$
- There are 15 girls and 25 boys in the class

- Rate:

- A rate typically refers to two quantities measured against each other.
- Common measures of rate include:
- Hourly rates of pay: \$/hr, Rs/hr

- Exchange rates: Rs/\$, \$/£
- Flow rates of liquids: l/hr
- Fuel consumption: l/km
- Other measures of rates include:
- Density: g/cm^3 , kg/cm^3
- Pressure: N/m^2 , N/m^3
- Population density: thousand of people/ km^2
- Speed: m/s, km/hr

- Time:

12-hour and 24-hour clocks:

- In 12-hour clocks, am or pm **must** be stated.
- In 24 hour clocks, the pm hours after 12 must be added to it.
- E.g 1:00 pm = 13:00
- Am or pm is **not** stated in 24-hour clocks
- Single-digit am times in 24-hour clocks are written with a **zero** before them.
- E.g 3:00 am = 03 00

Measures of Time:

- Time can be measured in seconds, minutes, hours etc.
- One second has 1000 milliseconds.
- One minute has 60 seconds.
- One hour has 60 minutes or 3600 seconds.
- One day has 24 hours.
- One week has 7 days.
- One month has 30 days.
- One year has 365 days, 12 months or 52 weeks.

- Money:

Currencies:

- Different countries have different currencies and different currencies have different values.
- Currencies are exchanged based on exchange rates.
- Currencies include: \$ Dollar (One dollar has 100 cents), Rs Rupees, £ Pounds.
- Simple and Compound Interest:
- Simple Interest:
- Simple interest shows the same growth every year.
- The formula for simple interest is:
- Simple Interest = $\frac{PRT}{100}$
- P = Principal Amount (Amount Invested)
- R = Interest Rate (The interest rate is always in a percentage, hence the product of PRT is divided by hundred)
- T = Time period (Usually in years)
- Compound Interest:

- Compound interest shows exponential growth.
- The formula for compound interest is:
- Compound Interest = $P(1 + r/100)^n$
- P = Principal Amount (Amount Invested)
- r = Interest Rate
- n = Time period (Usually number of years)

- Surds:

- Surds are square roots of imperfect squares
- E.g $\sqrt{27}$, $\sqrt{30}$

- Simplification of Surds:

- Multiplication of Surds:
 - $\sqrt{3} \times \sqrt{7} = \sqrt{(3 \times 7)} = \sqrt{21}$
- Division of Surds:
- $\sqrt{30}$ can also be written as, $\sqrt{3} \times \sqrt{10}$ or $\sqrt{60} \div \sqrt{2}$
- To completely simplify surds, the surd needs to be divided into two roots one of which is a perfect square.
- E.g: $\sqrt{40} = \sqrt{4} \times \sqrt{10} = 2 \times \sqrt{10} = 2\sqrt{10}$

ALGEBRA

Indices:

- Indices, also known as exponents or powers
- All the Properties of Indices **only** apply for **Multiplication** and **Division**.

- Rule 1

- When two same numbers are being multiplied, their powers are added.

$$a^n \times a^m = a^{n+m}$$

$$2^9 \times 2^4 = 2^{9+4} = 2^{13}$$

$$a^n \times b^n = (a \times b)^n$$

$$9^2 \times 4^2 = (9 \times 4)^2$$

$$\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$$

$$\frac{9^2}{4^2} = \left(\frac{9}{4}\right)^2$$

- When two same numbers are being divided, their powers are subtracted.

$$a^n \div a^m = a^{n-m}$$

$$2^9 \div 2^4 = 2^{9-4} = 2^5$$

- Shifting the place of a number from the numerator to the denominator or vice versa, will change the sign of its power
E.g $2^4 = 1/2^{-4}$
- Brackets with powers are solved by multiplying the power of every number within the bracket with the power on the bracket.

$$(a^m)^n = a^{m \times n}$$

$$(2^9)^4 = 2^{9 \times 4}$$

E.g $(2^4 \times 3^2)^2 = 2^8 \times 3^4$

- A number with the power **0** will always be equal to **1**.

$$x^0 = 1$$

$$8^0 = 1$$

→ A **fractional** power represents an **under root** and **cube root**.

$$\sqrt{x} = x^{\frac{1}{2}}$$

$$\sqrt[3]{x} = x^{\frac{1}{3}}$$

→ They can be solved in the following way as well:

$$\text{E.g } 41/2 = 2^2 \times \frac{1}{2} = 2^{\frac{2}{2}} = 2$$

→ The simplified answer must **always** contain **positive** indices.

Algebraic Manipulation

- In algebra, letters can be used to represent numbers.
- An algebraic expression is a combination of **constants**, **variables**, and **mathematical operations**.
- Example: **$3x + 5$**
- When solving questions of algebraic manipulation, **simplify** expressions by collecting like terms.
- Terms that have the **same** variables raised to the same power are called **Like Terms**.
- Combine like terms by adding or subtracting coefficients.
- For Example: **$2a^2 + 3ab - 1 + 5a^2 - 9ab + 4$**

$$= 7a^2 - 6ab + 3$$
- When asked to expand an expression, open the brackets by multiplication.
- For Example: **$(2x - 3)(x + 4)$**

$$2x(x + 4) - 3(x + 4)$$

$$2x^2 + 8x - 3x - 12$$

$$2x^2 + 5x - 12$$

Factorisation

- When the question asks for factorization, pick **common** factors
- For Example: $x^2 - 5y - xy + 5x$

$$x^2 - xy + 5x - 5y$$

$$x(x - y) + 5(x - y)$$

$$(x + 5)(x - y)$$

- Algebraic Fractions

- Questions often ask to manipulate algebraic fractions. These types of questions are solved by using methods like the

- Middle-term breaking,
 - Three Algebraic identities,
 - Quadratic formula.
 - Completing Square.
- For Example: $\frac{3a}{4}(\frac{9a}{10})$
 $\frac{27a}{40}$
- Rational expressions are simplified the following way:
- For Example: $\frac{x^2 - 2x}{x^2 - 5x + 6}$
 $\frac{x(x-2)}{x^2 - 3x - 2x + 6}$
 $\frac{x(x-2)}{x(x-3) - 2(x-3)}$
 $\frac{x(x-2)}{(x-2)(x-3)}$
 $\frac{x}{x-3}$

- Quadratic formula:

- Questions of completing the square are solved the following way:
- Example :Write $x^2 - 6x + 3$ in the form $(x-a)^2 + b$
 $x^2 - 2(x)(3) + (3)^2 - (3)^2 + 3$
 $(x-3)^2 - 9 + 3$
 $(x-3)^2 - 6$

Algebraic Equations

- Linear equation:

- For Example: $3x + 4 = 10$
 $3x = 6$
 $x = 6/3$
 $x = 2$

- Fractional Equations

- For Example: $\frac{10}{x} = x + 3$
 $10 = x^2 + 3x$
 $x^2 + 3x - 10 = 0$
 $x^2 - 2x + 5x - 10 = 0$
 $x(x-2) + 5(x-2) = 0$
 $(x+5)(x-2) = 0$
 $(x+5) = 0$ $(x-2) = 0$ gives 2
 $x = -5$ $x = 2$

Simultaneous Equations

- Substitution method:

- For Example: $3x + 4y = 3$
 $2x - y = 13$

- By substitution method,
- Multiply the second equation by 4 to equate the value of y
- Hence,
- $3x+4y=3$
- $8x-4y=52$
- Cancel the value of y and solve for the value of x and the value after '='
- $11x=55$
- $x=5$
- Substitute the value of x in either equation 1 or equation 2 of the question.
- $3(5)+4y=3$
- $4y=3-15$
- $y=-12/4$
- $y=-3$

- Changing the subject of the formula:

- **Identify the Target Variable:** Determine which variable you want to make the subject of the formula.
- **Isolate the Target Variable:** Move all other terms and variables to the opposite side of the equation.
- Use inverse operations to isolate the target variable.
- **Simplify:** Combine like terms and simplify the expression.

INEQUALITIES

- When representing and interpreting inequalities on a number line:
 - Open circles should be used to represent strict inequalities ($<$, $>$)
 - Closed circles should be used to represent inclusive inequalities ($\leq$, $\geq$)
- To solve linear inequalities, Treat the inequality like an equation and solve for x.
- Determine the sign based on the original inequality.
- Express the solution in interval or set notation.
- Represent and interpret linear inequalities in two variables graphically.
- The following conventions should be used:
 - Broken lines should be used to represent strict inequalities ($<$, $>$)
 - Solid lines should be used to represent inclusive inequalities ($\leq$, $\geq$)
 - shading should be used to represent unwanted regions (unless otherwise directed in the question).

PROPORTION

- Two quantities are in inverse proportion if an increase in one results in a proportional decrease in the other.
- Direct Proportion: $y=kx$
- Inverse Proportion: $y=k/x$

- Finding Unknowns: Use given values to determine the constant of proportionality (k) and then substitute to find unknown quantities.

- GRAPHS IN PRACTICAL SITUATIONS

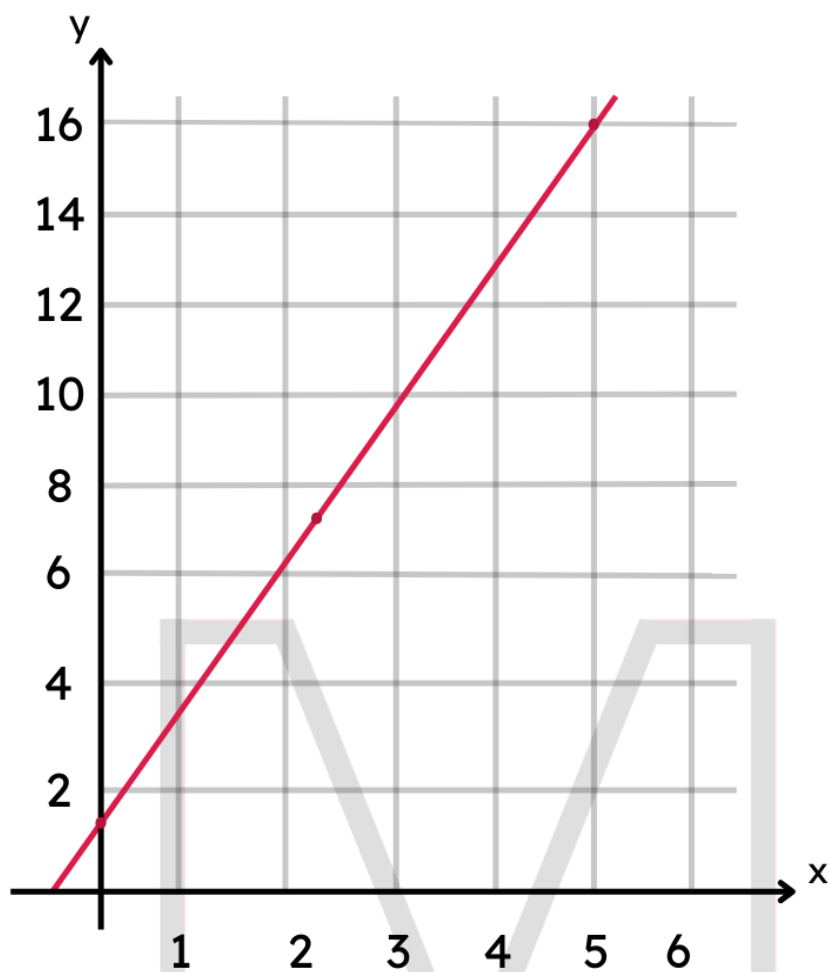
- Travel graphs: A travel graph represents the relationship between time and distance travelled. The gradient of the graph represents speed. Flat sections indicate the object is stationary.
- Conversion graphs: Conversion graphs show the relationship between two different units of measurement. Slope or gradient may represent a conversion factor. Points on the graph relate values in different units.
- Graphs from given data:
- Given data pairs (time, distance): (0, 0), (1, 50), (2, 100). Choose time on the x-axis and distance on the y-axis. Scale: 1 unit on the x-axis represents 1 hour, and 1 unit on the y-axis represents 50 km. Plot points (0, 0), (1, 50), (2, 100), and join them.
- Distance-time graph: gradient represents speed. Steeper gradient indicates higher speed. Speed-time graph: gradient represents acceleration or deceleration. Flat sections indicate constant speed. $\text{speed} = \text{distance} / \text{time}$
- Area under the s-t graph represents distance travelled. $\text{area} = \text{speed} \times \text{time}$

- GRAPHS OF FUNCTIONS

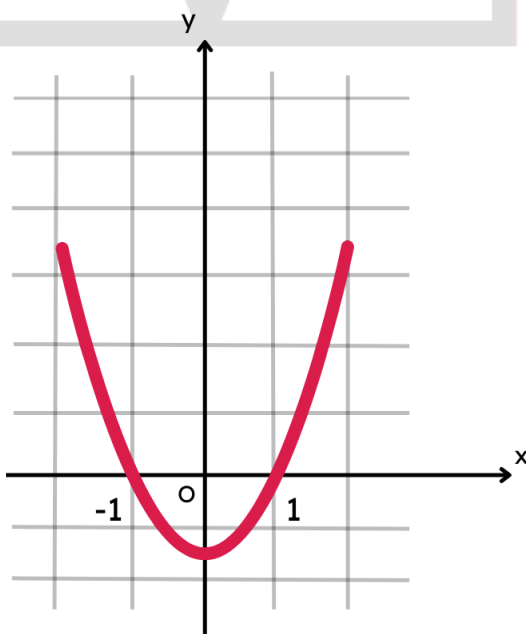
- ax^n : a =coefficient (rational number), n =exponent
- $abx+c$ = coefficients (rational numbers, b is a positive integer)
- When constructing tables, choose values for the independent variable. Use the function formula to calculate corresponding dependent variable values.
- When drawing graphs, choose a scale for both axes, plot points from the table, and join points with smooth lines or curves.
- When asked to recognize and interpret graphs, identify key features (intercepts, turning points) and explain the behaviour of the function based on the graph.
- Exponential growth
- $y=ab^x$ where $b > 1$
- Exponential decay
- $y=ab^x$ where $0 < b < 1$
- To estimate the gradient of a curve, draw a tangent at specific points on the curve and the gradient of the tangent will represent the gradient of the curve.

- SKETCHING CURVES

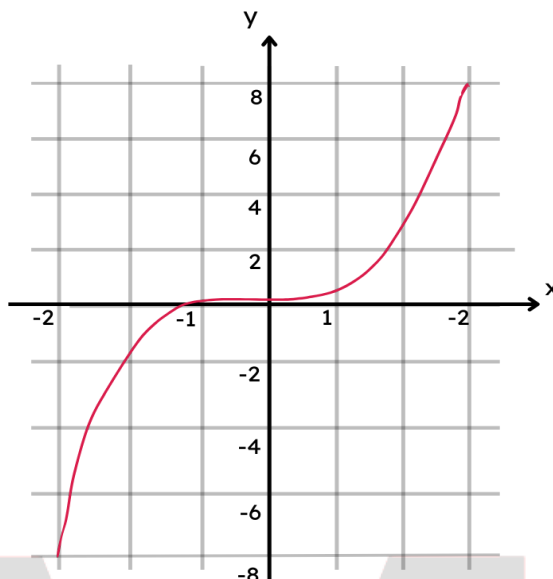
- Linear graphs (e.g. $ax+by=c$): A straight line on the Cartesian plane. The slope (m) represents the rate of change. The y-intercept is the value of y when $x=0$



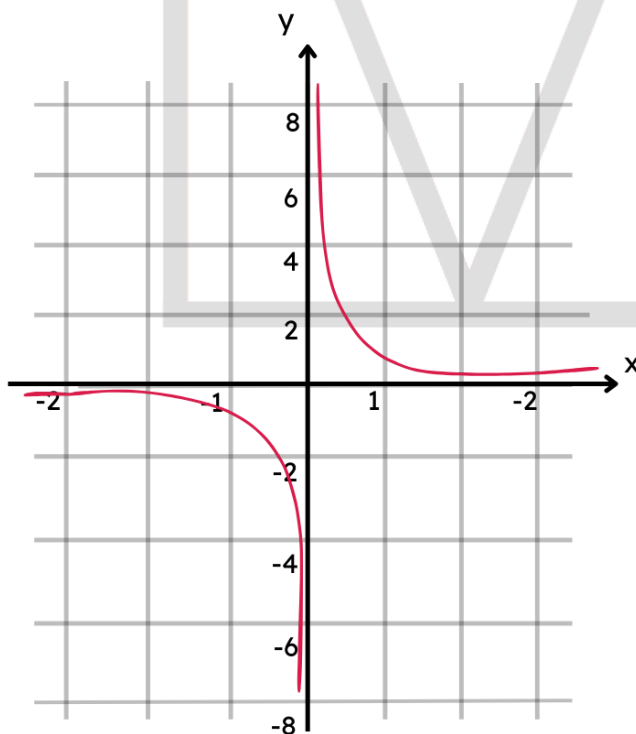
→ Quadratic graphs (e.g. $y=ax^2+bx+c$): A parabola, either opening upwards or downwards. The coefficient a determines the direction and width of the parabola.



- Cubic graphs (e.g. $y=ax^3+d$): A curve with one or two inflection points. The shape and direction of the curve are determined by the signs of the coefficients.



Reciprocal functions (e.g. $y=\frac{a}{x}$): A hyperbola, with two branches asymptotic to the x and y-axes. The graph approaches the x and y-axes but never intersects them.



- Exponential function (e.g. $y=ab^x$): An increasing or decreasing curve, depending on the base
- To sketch graphs, determine the form of the function and locate intercepts, turning points, asymptotes, etc. Select points that help visualise the graph accurately. Use the

chosen points to sketch the curve. Label the x and y-axes, and mark key points on the graph.



- FUNCTIONS

- A function f is a relation between a set of inputs (domain) and a set of possible outputs (range), such that each input is related to exactly one output.
- **Domain:** The set of all possible input values for a function.
- **Range:** The set of all possible output values for a function.
- The inverse function f^{-1} “undoes” the work of f
- Composite functions are formed by applying one function to the result of another.

GEOMETRY

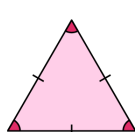
Polygon:

→ A polygon is a flat (plane) shape with n straight sides

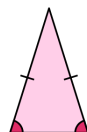
Polygon	Sides
Triangle	3
Quadrilateral	4
Pentagon	5
Hexagon	6
Heptagon	7
Octagon	8
Nonagon	9
Decagon	10

- Types of Triangles:

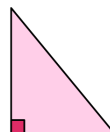
- There are three types of triangles: Equilateral, isosceles, and right triangle
- Equilateral: 3 equal sides and 3 equal angles
- Isosceles: 2 equal sides and 2 equal angles
- Right: has one 90° angle



Equilateral Triangle



Isosceles Triangle

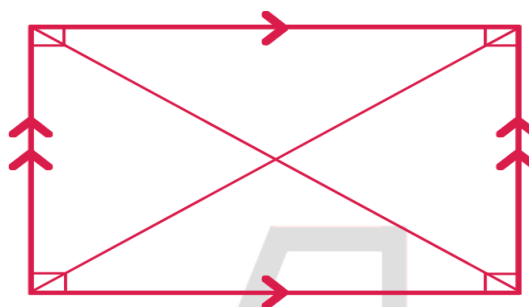
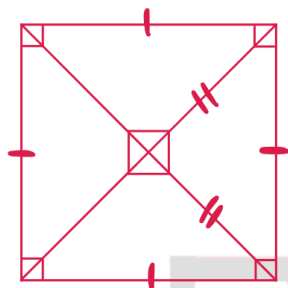


Right-angled Triangle

Properties of Quadrilaterals

- Square:

- All four sides are equal
- All sides intersect at right angles
- Diagonals intersect at midpoints of each other
- Diagonals intersect at right angles
- Diagonals are equal

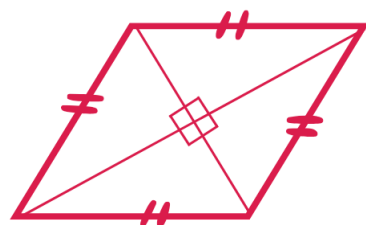


- Rectangle:

- Opposite sides are equal and parallel
- Sides intersect at right angles
- Diagonals intersect at midpoints of each other
- Diagonals do not intersect at right angles
- Diagonals are equal

- Rhombus:

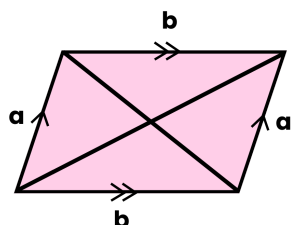
- All sides are equal
- Sides do not intersect at right angles
- Diagonals intersect at midpoints of each other
- Diagonals intersect at right angles
- Diagonals are not equal



- Parallelogram:

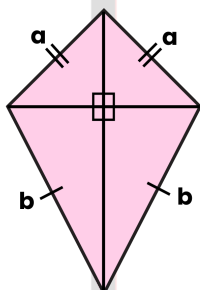
- Opposite sides are equal and parallel
- Sides do not intersect at right angles
- Diagonals intersect at midpoints of each other
- Diagonals do not intersect at right angles

→ Diagonals are not equal



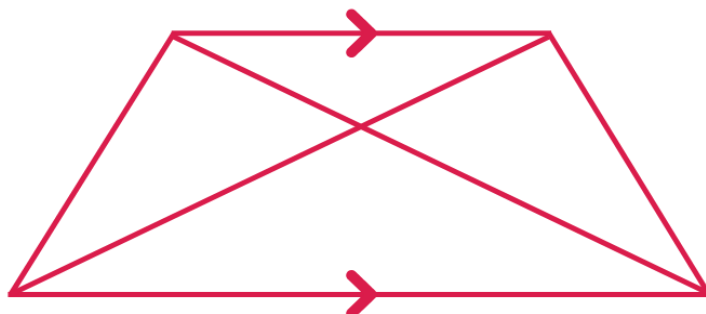
- Kite:

- Adjacent sides are equal
- Sides do not intersect at right angles
- Diagonals do not intersect at midpoints of each other
- The longer diagonal intersects the short diagonal at midpoints
- Diagonals intersect at right angles
- Diagonals are not equal



- Trapezium:

- One pair of sides are parallel
- Sides do not intersect at right angles
- Diagonals do not intersect at midpoints of each other
- Diagonals are not equal
- Diagonals do not intersect at right angles



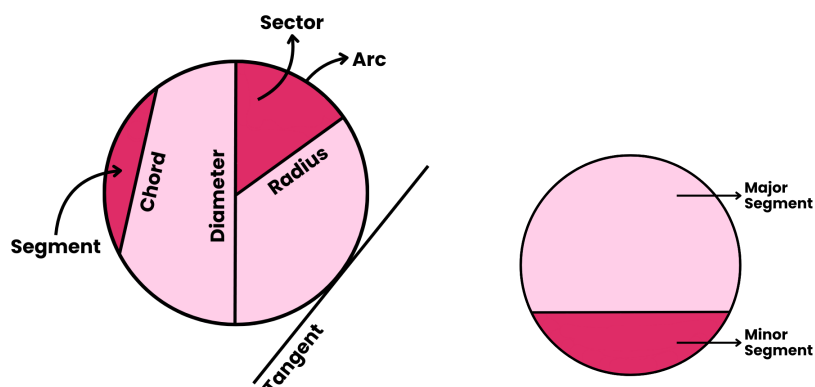
- 3D Shapes:

- 3D shapes have a number of faces, vertices and edges
- A face is an individual surface of the 3D shape
- A vertex (plural, vertices) is a corner of the 3D shape
- An edge joins one vertex to another

Shape	Faces	Edges	Vertices
Cube	6	12	8
Cuboid	6	12	8
Cylinder	3	2	0
Triangular Prism	5	9	6
Square – Based Prism	5	8	5
Tetrahedron	4	6	4

- Properties of a Circle:

- Centre is a point from which the circle is drawn
- Circumference is the perimeter of the circle
- Arc is part of a circle
- Radius is a line drawn from the centre of the circle till the circumference
- Chord is a line that touches the circumference twice. It divides the circle into two parts; minor segment & major segment
- Diameter is a chord that passes through the centre of the circle. It divides the circle into two equal halves
- Tangent is a line that touches the circumference of the circle just once. The point where the radius and tangent intersect make a 90°
- Segment is the area bounded by the chord and the arc length
- Sector is a part of a circle. Two radii and the arc bound it.



Bearings:

- Bearings are a way of describing and using directions as angles
- There are three rules which must be followed every time a bearing is defined
 - They are measured from the North direction
 - They are measured clockwise.
 - The angle should always be written with 3 figures

- Steps to answer a question involving drawing bearings:

- **STEP 1:** Draw a diagram adding in any points and distances you have been given
- **STEP 2:** Draw a North line (arrow pointing vertically up) at the point you wish to measure the bearing from
- If you are given the bearing from A to B draw the North line at A
- **STEP 3:** Measure the angle of the bearing given from the North line in the clockwise direction
- **STEP 4:** Draw a line and add the point B at the given distance
- Knowing the compass directions for the common directions is helpful
 - Due east means on a bearing of 090°
 - Due south means on a bearing of 180°
 - Due west means on a bearing of 270°
 - Due north means on a bearing of 360° (or 000°)
 - Due Northeast means on a bearing of 045°
 - Due Southeast means on a bearing of 135°
 - Due Southwest means on a bearing of 225°
 - Due Northwest means on a bearing of 315°

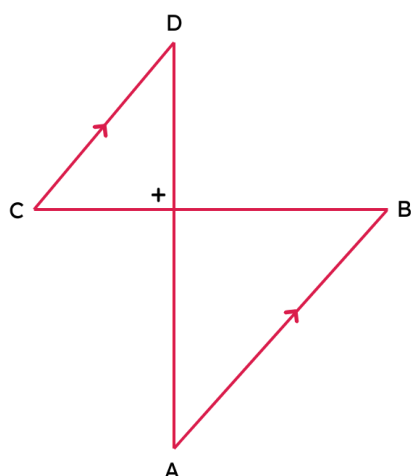
Similarity:

- Two shapes are similar if they have the **same** shape and their corresponding sides are in **proportion**
- If two triangles of different sizes have the same angles, they are **similar**.
- To show that two non triangular shapes are similar you need to show that their corresponding sides are in proportion
- Divide the length of one side by the length of the same side on the other shape to find the scale factor
- If the scale factor is the same for all corresponding sides, then the shapes are similar
- If two shapes are similar, then the following formula can be applied:

$$\frac{\text{Volume of A}}{\text{Volume of B}} = \frac{(\text{length of A})^3}{(\text{length of B})^3}$$

- Example:

- In the diagram below, AB and CD are parallel lines.
- Show that triangles ABX and CDX are similar.



- Angle $AXB = \text{angle } CXD$ (Vertically opposite angles are equal)
- Angle $ABC = \text{angle } BCD$ (Alternate angles on parallel lines are equal)
- Angle $BAD = \text{angle } ADC$ (Alternate angles on parallel lines are equal)
- All three corresponding angles are equal, so the two triangles are similar

Symmetry:

- Symmetry is of two types
- Line (or Plane) symmetry which deals with reflections and mirror images of shapes or parts of shapes in both 2D and 3D
- Rotational symmetry which deals with how often a shape looks identical (congruent) when it has been rotated

- Rotational symmetry

- refers to the number of times a shape looks the same as it is rotated 360° about its centre
- This number is called the order of rotational symmetry

- Example:

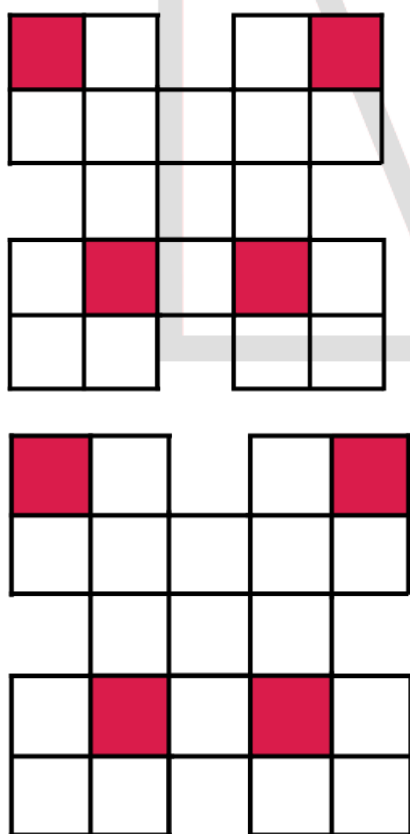
- Write down the order of rotational symmetry of the following shape.



→ The order of Rotational Symmetry is 2.

- Example:

→ For the shape below, shade exactly 4 more squares so that the shape has rotational symmetry order 4.



- Line symmetry

- Lines of symmetry can be thought of as a folding line too
- Folding a shape along a line of symmetry results in the two parts sitting exactly on top of each other

- Planes of symmetry

- A plane of symmetry is a plane that splits a 3D shape into two congruent (identical) halves
- All prisms have at least one plane of symmetry
- Cubes have 9 planes of symmetry
- Cuboids have 3 planes of symmetry
- Cylinders have an infinite number of planes of symmetry
- The number of planes of symmetry in other prisms will be equal to the number of lines of symmetry in its cross-section plus 1
- Pyramids can have planes of symmetry too
- The number of planes of symmetry in other pyramids will be equal to the number of lines of symmetry in its 2D base
- If the base of the pyramid is a regular polygon of n sides, it will have n planes of symmetry

- Basic Angle Properties:

- Angles that meet at a point add up to 360°
- Angles that meet at a point on a straight line add up to 180°
- The three interior angles inside any triangle add up to 180°
- The four interior angles inside any quadrilateral add up to 360°
- Vertically opposite angles are equal

- Angles in a polygon:

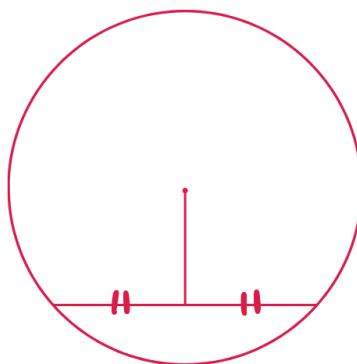
- To find the sum of the interior angles in a polygon of n sides, use the rule
- **Sum of Interior Angle = $180^\circ \times (n - 2)$**
- The sum of the exterior angles in any polygon always add up to 360°

- Angles in Parallel lines:

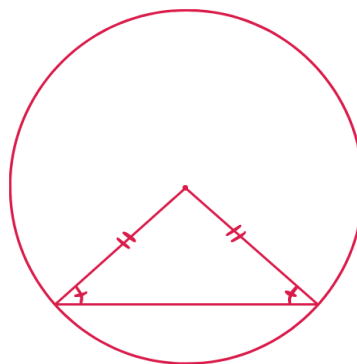
- Corresponding angles are equal: A line cutting across two parallel lines creates four pairs of equal corresponding angles, as shown in the diagram below:
- Alternate angles are equal: A line cutting across two parallel lines creates two pairs of equal alternate angles, as shown in the diagram below:
- Allied (co-interior) angles add to 180° : A line cutting across two parallel lines creates two pairs of co-interior angles

- Angle properties of a circle:

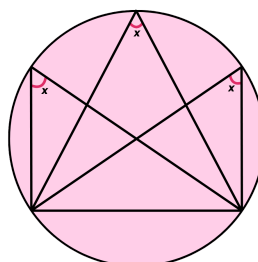
- 90° made from a centre on a chord cuts the chord in two equal halves



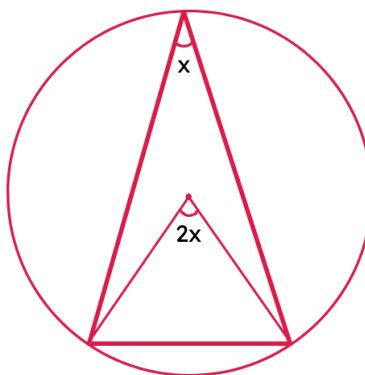
→ If we join ends of a chord to centre, it makes an isosceles triangle



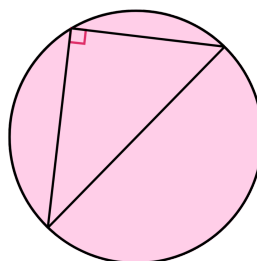
→ Angle made on circumference from ends of same chord are always equal in the same segment



→ Angle made on centre is double the angle made on circumference if they start from the same chord



→ Angle made at circumference starting from ends of a diameter is always 90°

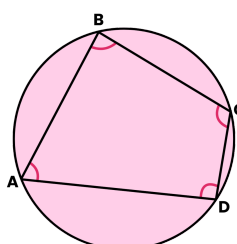


→ Cyclic Quadrilateral (all ends must be touching circumference)

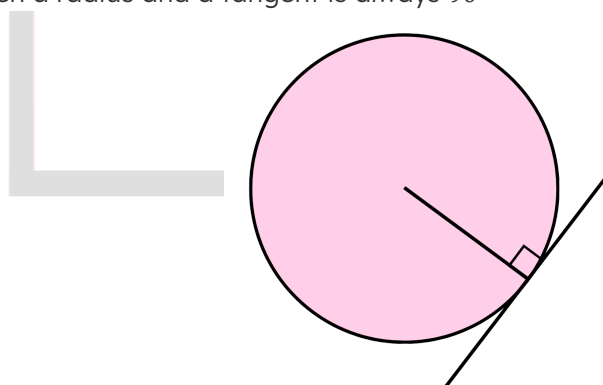
→ Sum of opposite angles = 180°

→ $A + C = 180^\circ$

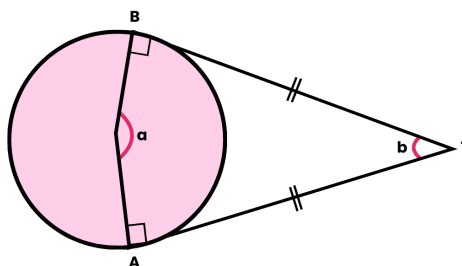
→ $B + D = 180^\circ$



→ Angle between a radius and a tangent is always 90°



→ Tangent kite



STATISTICS

CLASSIFYING STATISTICAL DATA

- **Statistical Data:** Information collected and analysed to draw conclusions, make predictions, or inform decision-making.
- **Classification:** Organising data into categories or groups based on common characteristics.
- **Categorical Data:** Qualitative data that falls into distinct categories.

- Interpreting Statistical Data

- While reading tables, identify column and row headers, and then interpret values within the table cells. Analyse patterns, trends, or relationships within the data.
- Make inferences based on the information presented.

- Types of statistical diagrams:

- **Histograms, bar charts, line graphs, pie charts, scatter plots, etc.**
- While reading and interpreting, identify labels, scales, and axes, and analyse the distribution, trends, or relationships depicted.
- Draw conclusions based on the patterns or trends observed. Consider the context and implications of the data.

- Comparing data:

- In tables, compare values directly within tables.
- In graphs, compare trends, patterns, or distributions visually.
- While making statistical measures, compare summary statistics such as mean, median, mode, range, standard deviation, etc.

- Limitations in data analysis:

- Sampling bias: The sample may not be representative of the population.
- Data quality issues: Incomplete, inaccurate, or biased data.
- *Question assumptions, consider alternative explanations, and acknowledge uncertainties when analysing data.

- Averages and Means spread

- **Mean:** The arithmetic average calculated by summing all values and dividing by the number of values.
- **Median:** The middle value in a sorted dataset. If there's an even number of values, it's the average of the two middle values.
- **Mode:** The value that appears most frequently in the dataset.
- **Range:** The difference between the maximum and minimum values in the dataset.

- Estimating mean of grouped data:

- **Grouped data:** Data organised into intervals or groups rather than individual values.
- **Midpoint Method:** Approximate the values by using the midpoints of each interval.
- **Frequency Method:** Weight the midpoints by the frequency of each interval.

- Estimating modal class from grouped frequency distribution:

- **Modal class:** The interval or class with the highest frequency in a grouped frequency distribution.
- To identify modal class, Look for the interval with the highest frequency.

Statistical Charts And Diagrams

- Bar charts:

- Bar charts represent data using rectangular bars where the length of each bar is proportional to the value it represents.
- To construct a bar chart, Draw horizontal or vertical bars for each category or group. Ensure equal width and spacing between bars.
- Bar charts can be interpreted by comparing the heights of the bars to understand relative quantities.
- Identify trends, patterns, or comparisons between categories.

- Pie charts:

- Pie charts represent data as a circle divided into sectors, with each sector representing a proportion of the whole.
- To construct a pie chart, divide the circle into sectors proportional to the data values.
- Label each sector with the corresponding category or value.
- Pie charts can be interpreted by comparing the sizes of the sectors to understand the distribution of the data.
- Highlight the relative proportions of different categories or groups.

- Pictograms:

- Pictograms use pictures or symbols to represent data, with each picture representing a certain quantity or value.
- To construct a pictogram, choose a consistent symbol to represent each unit or value. Draw a number of symbols corresponding to the data value.
- Pictograms can be interpreted by counting the number of symbols to determine the value represented.
- Compare the sizes or quantities represented by different symbols.
- **Simple frequency distributions:**
 - Simple frequency distributions represent the frequency of occurrence of each value or range of values in a dataset.
 - Construction of these can be done by listing the distinct values or intervals of values.

- Record the frequency of occurrence for each value or interval.
- These can be interpreted by analysing the frequency distribution to identify the most common values or intervals.
- Visualise the distribution using histograms or bar charts to understand the spread of the data.

SCATTER DIAGRAMS

- Scatter diagrams, also known as scatter plots, are graphical representations of paired data points, where each point represents the values of two variables.
- To construct a scatter diagram, Plot each pair of data points on a Cartesian plane, with one variable on the x-axis and the other on the y-axis.
- To interpret, examine the pattern or distribution of points to identify relationships between the variables. Look for trends, clusters, or outliers in the data.
- **Positive Correlation:**
 - A positive correlation exists when an increase in one variable is associated with an increase in the other variable.
 - In a scatter diagram, points tend to cluster around a line sloping upwards from left to right.
- **Negative correlation:**
 - A negative correlation exists when an increase in one variable is associated with a decrease in the other variable.
 - In a scatter diagram, points tend to cluster around a line sloping downwards from left to right.
- **Zero correlation:**
 - A zero correlation exists when there is no apparent relationship between the variables.
 - In a scatter diagram, points are scattered randomly without any discernible pattern.
- **Drawing by eye:**
 - Draw a straight line that closely follows the general trend or pattern of the data points.
 - The line should pass through the centre of the data points.
 - To interpret, Use the line of best fit to estimate values or make predictions about the relationship between the variables.

CUMULATIVE FREQUENCY DIAGRAMS

- A cumulative frequency table shows the running total of frequencies up to each data value.
- To construct, list the data values in ascending order. Record the frequency of each value.
- Calculate the cumulative frequency by adding up the frequencies as you go down the table.

- Estimating Median:

- Locate the middle of the cumulative frequency diagram. Read the corresponding value on the x-axis.
- Percentiles: Divide the cumulative frequency scale into 100 equal parts.
- Read the corresponding value on the x-axis for the desired percentile.
- Quartiles: Divide the cumulative frequency scale into 4 equal parts.
- Read the corresponding values on the x-axis for each quartile.
- -Q1 represents the 25th percentile (lower quartile).
- -Q2 represents the median (50th percentile).
- -Q3 represents the 75th percentile (upper quartile).
- Interquartile range: $IQR = Q3 - Q1$

Histogram

- **Histogram:** A graphical representation of the frequency distribution of continuous data.
- It consists of adjacent rectangles, where the area of each rectangle represents the frequency of a class interval.

- Class Intervals:

- Divide the range of the data into equal intervals.
- Choose appropriate intervals to display the data effectively.
- **Height of Bars:** The height of each bar represents the frequency or frequency density of the corresponding class interval.
- Frequency density is calculated as frequency divided by the width of the interval.
- **Frequency Density:** The frequency per unit interval width.
- **Calculation:** Frequency density = $\frac{\text{frequency}}{\text{interval width}}$
- It is used when the intervals have different widths, ensuring that the area of each bar represents the frequency accurately.

Mensuration

- Units of Measure:

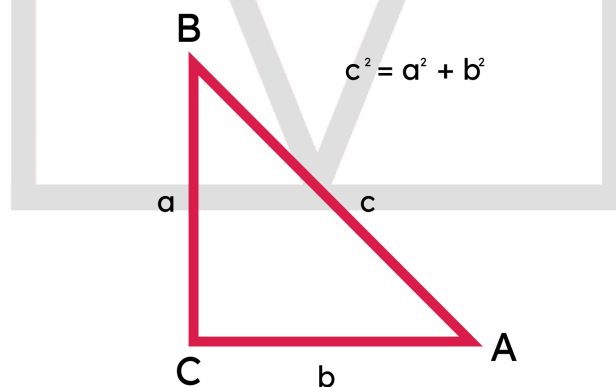
Quantity	Units
Mass	Grams (g), Kilograms (kg)
Length	Millimeter (mm), Centimeter (cm), Meter (m), Kilometer (km)
Area	Square Millimeter (mm ²), Square centimeter (cm ²), Square meter (m ²)
Volume	Cubic Millimeter (mm ³), Cubic centimeter (cm ³), Cubic meter (m ³)
Capacity	Milliliter (ml), Litre (l)

Trigonometry

Pythagoras Theorem:

→ For all the right-angled triangles:

$$h^2 = p^2 + b^2$$



- Trigonometric Ratios of Acute Angles:

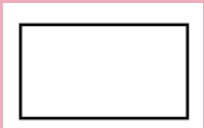
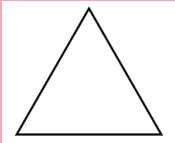
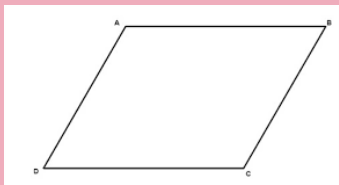
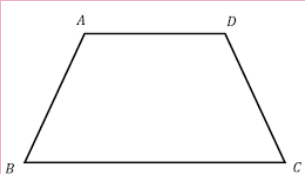
→ All angles in a right angle triangle are acute. (Between 0 to 90°).

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$

- Area and Perimeter:

	Shape	Area	Perimeter
Rectangle		$\ell \times w$	$2(\ell + w)$
Square		ℓ^2	4ℓ
Triangle		$\frac{1}{2}bh$	Sum of all 3 sides
Parallelogram		$b \times h$	$2(a + b)$
Trapezium		$\frac{1}{2}(a + b) \times h$	Sum of all sides

Circles, Arcs & Sectors:

The circumference of a circle is its perimeter or its arc length, as if it opened up and straightened out to a line segment.

Formulae:

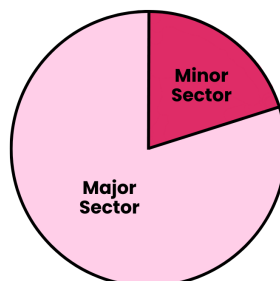
Circumference of circle: $2\pi r$

Area of circle: πr^2

Area of semi- circle: $\frac{1}{2}\pi r^2$

Perimeter of semi- circle: $2\pi r/2$

- Arc lengths and Sectors are fractions of the circumference and area of a circle.
- Major sectors are those which have a central angle greater than 180 degrees.
- Minor sectors are those which have a central angle less than 180 degrees.



$$A = \frac{1}{2}ab \sin C$$

$$A = \frac{1}{2}bc \sin A$$

$$A = \frac{1}{2}ac \sin B$$

Sine Rule:

- It can be used when:
- Two angles and one side given.
- Two sides and one non-included angle given.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

OR

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule:

- It can be used when:
- Three sides are given.
- Two sides and one angle is given.

$$a^2 = b^2 + c^2 - (2bc \cos A)$$

$$b^2 = a^2 + c^2 - (2ac \cos B)$$

$$c^2 = a^2 + b^2 - (2ab \cos C)$$

Matrices, Vectors & Transformation

Matrices (2024 only)

- A matrix is a rectangular array of numbers arranged in rows (horizontal) and columns (vertical).
- Matrices of the same order can be added or subtracted by adding or subtracting **corresponding** elements.
- The **product** of two matrices is computed by multiplying **rows** of the **first** matrix by **columns** of the **second** matrix.

- Calculate the Product of a Matrix and a Scalar Quantity:

- Multiply each element of the matrix by the scalar quantity.
- This operation scales the entire matrix by the scalar factor.

- Use the Algebra of 2x2 Matrices Including Zero and Identity 2x2 Matrices:

- A 2x2 matrix is a matrix with two rows and two columns.
- The zero matrix (or null matrix) is a matrix in which all elements are zero.
- The identity matrix (usually denoted by I) is a special square matrix where all diagonal elements are ones and all off-diagonal elements are zeros.

- Calculate the Determinant and Inverse of a Non-Singular Matrix:

- The determinant of a square matrix is a scalar value that can provide information about the matrix's properties and its solvability in systems of linear equations.
- The determinant of a non-singular matrix is calculated by summing products of elements along diagonals with alternating signs.
- The inverse of a non-singular matrix A (usually denoted by A^{-1}) is another matrix such that their product yields the identity matrix ($A \times A^{-1} = I$).
- The inverse of a non-singular matrix is found by dividing the adjugate matrix by the determinant of the original matrix.

Transformations

- Transformations are movements and changes of shape according to given rules.
- There are two types of transformations:

- Isometric Transformation:

- Shape and size remain the same but position/orientation changes.
- Reflection, Rotation and Translation are isometric transformations.

- Non-Isometric Transformation:

- Shape/size and position changes.
- Enlargement is a non- isometric transformation.

Reflections

- A reflection, or flip, is a transformation that creates symmetry on the coordinate plane. You can use matrix multiplication to graph reflections in the coordinate plane.
- Under reflection, an object and image are symmetrical about the mirror line. The object and image are the same shape and size.
- Reflection does not preserve orientation. The object and image face in opposite directions.
- Points on the mirror line are invariant. The image is as far behind the mirror line as the object is in front of it.

- How to describe a reflection:

- Write the name of transformation (Reflection).
- Write the Equation of line of reflection.

- How to draw a reflection:

- Select a point on the object and measure its distance (or count squares) from the line of reflection.
- The distance must be measured in the perpendicular direction to the mirror line.
- Measure the same distance (or count squares) beyond the mirror line and mark the point.
- Repeat the same procedure with all other points and join all new points to get the image.

- How to find the line of reflection:

- Draw the line bisectors of two corresponding points of object and image. Both line bisectors overlap which shows that it is reflection otherwise it is rotation.

Rotation:

- A rotation is a transformation that turns a figure about a fixed point called a centre of rotation.
- Rotation is defined by its centre, angle, and direction. The centre of rotation is the only invariant point.

- How to identify rotation:

- The object and image have the same shape and size.
- The object and image are facing different ways (but not flipped as in reflection)
- All points on the object and image are at the same distance from the centre of rotation.

- How to describe the rotation:

- Write down the name of transformation (Rotation)
- Write the coordinates of centre of rotation, angle, and direction of rotation.

- How to rotate an object by using tracing paper:

- Place tracing paper on the object.
- Trace one the axes and object on tracing paper.
- Place the point of the pencil on the centre of rotation.
- Turn the tracing paper through the given angle and direction. The traced axes will guide here. ie. Mark the image from tracing paper on original paper.

- How to rotate an object geometrically:

- Join the centre of rotation with one of the points of the object by a dotted straight line.
- Place the centre of the protractor on the centre of rotation and zero towards the point.
- Rotate the dotted line by marking the given angle in given direction (Clockwise or Anticlockwise).
- Measure the distance between the centre of rotation and selected point on the object ie. Mark the point at the equal distance from the centre of rotation on the rotated dotted line.
- Repeat the same procedure for the rest of the points.
- Join all the new marked points to get a rotated image.

- How to find the centre of rotation:

- Draw line bisectors of two pairs of corresponding points of Object and image. The point of intersection of the two line bisectors is the centre of rotation.

- How to find angle and direction of rotation:

- Join corresponding points of object and image with centre of rotation and observe angle and direction.

Translation:

- A translation moves all points of an object on a plane the same distance and in the same direction.
- Translation has no invariant point.

- How draw a translation:

- Select one point on the object.
- Count horizontally the number of units shown on the top of the column vector.
- Then count vertically the number of units shown at the bottom of the column vector.
- Repeat the same procedure for all points.
- Join new marked points to get an image.

- How to identify a translation:

- The object and image have the same shape and size.
- The direction (orientation) of the object and image remains the same.

- How to describe a translation:

- Write the name of transformation (Translation).
- Write down column vectors (ab). The top number (a) shows horizontal displacement and bottom number (b) shows vertical displacement.

Enlargement

- Enlargement makes an object larger or smaller according to a given scale factor.
- Enlargement is defined by its centre and scale factor. A scale factor is the ratio of length of the image to the corresponding length of the object.
- If the scale factor is positive, then the object and image will be on one side of the centre of enlargement.
- If the scale factor is negative, then the centre of enlargement will be in between object and image.
- Centre of enlargement is the only invariant point.
- Scale Factor (k): $k =$
- Length of image
- length of object
- $\text{Area of Image} = k^2 \times \text{area of object}$

- How to identify an enlargement:

- The image is smaller or larger than the object.
- The shape and angles of the image and objects remain the same, but sides are proportional.
- The position of the image depends upon the scale factor.
- If the scale factor is positive, then the object and image is positioned on one side of the centre of enlargement.
- image orientation remains the same.
- If the scale factor is negative, then the centre of enlargement is positioned in between object and image.
- The image is also flipped.

- How to find centre of enlargement:

- Join the corresponding points on object and image by straight dotted lines.
- These lines intersect at a common point which will be the centre of enlargement.

- How to find scale factor:

- The scale factor is the ratio of length of image to the corresponding length of the object.
- $S.F =$

- length of image
- length of object
- Mention the sign (+ or -) of scale factor carefully.

- How to describe an enlargement:

- Write the name of transformation (Enlargement)
- Write coordinates of the centre of enlargement.
- Write scale factor.

- How to draw an enlargement geometrically:

- Join the centre of enlargement to each of the corners of the object by dotted line.
- If the S.F is positive, extend the lines in the direction of the object.
- If the S.F is negative, extend the lines in the opposite direction of the object.
- Measure the distance between the centre and corner of the object.
- Multiply this distance by scale factor and mark new distance from centre on the dotted line.
- Repeat the same procedure for the rest of the corners and mark new points.
- Join all new points to get an image.

Vectors

Vectors in two dimensions

- Describing a translation using a vector:

- Translation in mathematics refers to the process of moving points or shapes from one position to another in a coordinate system.
- A translation can be described using a vector represented by its components in terms of x and y coordinates.
- For example, if we have a vector v represented by (x,y) , it means that a point will be moved x points horizontally and y points vertically.

- Adding and Subtracting Vectors:

- Vectors can be added or subtracted component-wise.
- Addition of two vectors v and w results in a new vector $v+w$ where the corresponding components of v and w are added together.
- Subtraction of two vectors v and w results in a new vector $v-w$ where the corresponding components of w are subtracted from the components of v .

- Multiplying a Vector by a Scalar:

- Multiplying a vector by a scalar involves scaling the magnitude of the vector.
- If v is a vector and c is a scalar, then the product of cv is a new vector where each component of v is multiplied by c .
- This operation either stretches or compresses the vector depending on whether the scalar is greater than or less than 1, respectively.

- If the scalar is negative, the direction of the vector is reversed.

- Magnitude of a Vector

- The magnitude of a vector v in a two-dimensional space, represented as (x,y) , can be calculated using the Pythagorean theorem.
- The formula to calculate the magnitude is $|v| = \sqrt{x^2+y^2}$
- Here x and y are the components of the vector along the horizontal and vertical axes, respectively.

Vector Geometry

- Represent Vectors by Directed Line Segments:

- Vectors can be represented visually by directed line segments with an arrow indicating both direction and magnitude.
- The length of the line segment represents the magnitude of the vector, and the direction of the arrow indicates its direction.

- Use Position Vectors:

- Position vectors are vectors that represent the position of a point relative to a reference point, usually the origin.
- They are commonly denoted by r and expressed as the displacement from the origin to the point, typically in terms of Cartesian coordinates (x,y) .

- Use Sum and Difference of Vectors:

- Vectors can be combined using vector addition or subtraction.
- Vector addition involves adding the corresponding components of two or more vectors to find their resultant vector.
- Vector subtraction involves subtracting the components of one vector from another to find the resultant vector.
- By using vector addition or subtraction, any given vector can be expressed as the sum or difference of two or more coplanar vectors.

- Use Vectors to Reason and Solve Geometric Problems:

- Vectors provide a powerful mathematical tool for reasoning and solving geometric problems.
- They can be used to describe positions, displacements, velocities, forces, and more in a geometric context.
- Geometric problems involving angles, distances, and shapes can often be solved more efficiently using vector methods, particularly in three-dimensional space.

Probability

Simple Probability:

Probability Scale:

- The probability scale ranges from 0 to 1.
- Probability is written in the form of fractions.
- $\frac{1}{5}$ - The event has a one in five chance of happening
- Probability can also be written in the form of decimals or percentages:.
- $25\% = 0.25 = \frac{1}{4}$

- Probability Notation:

- Probability is noted in the following manner: $P(\text{Event}) = x$
Example:
- A die has 6 sides, each with a different number from 1-6
- The probability of the dice landing on 2 is:
- $P(2) = \frac{1}{6}$
- The probability of the dice landing on an even number is:
- $P(\text{Even number}) = \frac{1}{2}$
- Calculating Probability:
- $\text{Probability}(\text{Event}) = \frac{\text{Favourable Outcomes}}{\text{Total Outcomes}}$
- $1 - \text{Probability of an event occurring} = \text{Probability of the event not occurring.}$

Frequencies

- Frequency refers to the rate at which something occurs over a particular period of time:

- Frequency and Probability:

- In probability, frequency refers to the amount of times an event occurs.
- Frequencies are usually written to write a large number of outcomes in a smaller space.
- The relative frequency can be calculated by dividing the frequency of an outcome by the total number of outcomes or the sum of all the frequencies.
- The expected frequency can be calculated by multiplying the probability of an outcome by the number of trials.

Probability of Combined Events:

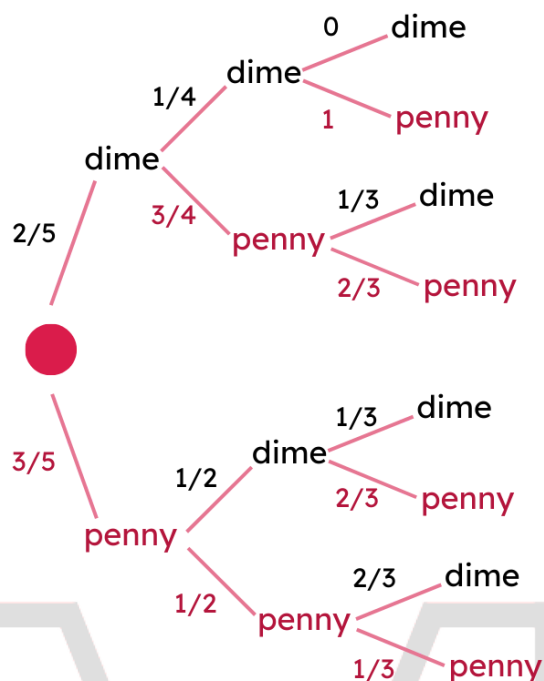
- Sample Space Diagrams:

- Sample space refers to all the possible outcomes of an experiment.
- Sample space diagrams are used to calculate probabilities of combined events.
- There are different types of sample space diagrams involving different operations.
- A sample space diagram is drawn as shown

+	1	2	3
7	8	9	10
8	9	10	11
9	10	11	12

- Tree Diagrams:

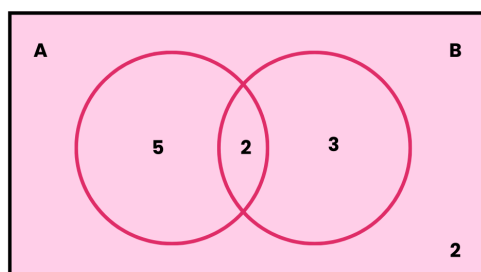
- The probability of each event happening is written along each branch.
- The event is written at the end of each branch.
- Tree diagrams depend on whether the experiment is carried out with or without replacement.
- If the experiment is carried out with replacement, the probability will not change with each sub-branch.
- If the experiment is carried out without replacement, the probability will change with each sub-branch.
- To calculate the probability of the sub-branch, both the favourable outcomes and the total number of outcomes should be subtracted by 1.
- The favourable outcomes should only be subtracted by one at the sub-branch that shows the probability of the same event occurring.
- Tree diagrams are drawn in the following manner:



- The probability of picking a dime in the first pick is = $\frac{2}{5}$
- The probability of picking a dime in the second pick is = $\frac{1}{4}$
- The probability of picking a dime in both the picks is = $\frac{2}{5} \times \frac{1}{4} = \frac{2}{20}$
- The probability of picking a dime in the first pick and a penny in both the second and third pick is = $\frac{2}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{12}{60}$

- Venn Diagrams:

- Each circle shows the probability of an event happening.
- The intersection of two circles shows the probability of both the events happening at the same time.
- The number outside both circles indicates the probability that none of the events take place.
- Probability Venn Diagrams are drawn as shown:



- A and B are two events.
- The Probability of Event A occurring is $= 5 + 2 = 7$
- The Probability of Event B occurring is $= 3 + 2 = 5$
- The Probability of both the events happening is $= 2$
- The Probability of neither event taking place is $= 2$

A Note from Mojza

These notes for Mathematics (4024/0580) have been prepared by Team Mojza, covering the content for O Level/ IGCSE 2024-27 syllabus. The content of these notes has been prepared with utmost care. We apologise for any issues overlooked; factual, grammatical or otherwise. We hope that you benefit from these and find them useful towards achieving your goals for your Cambridge examinations.

If you find any issues within these notes or have any feedback, please contact us at support@mojza.org.

Acknowledgements

Authors:

Abdul Haleem
Mian Abu Bakr Nadir
Alishba Ahmad
Sameen Maqsood
Sarim Abdullah

Proofreaders:

Abdul Haleem
Aroosham Mujahid
Miraal Omer

Designers:

Sarah Binte Sajid Khurshid
Abdul Haleem



MOJZA

4024 & 0580

O Levels & IGCSE

MATHEMATICS

FULL SYLLABUS NOTES

These notes are made to encompass the complete syllabus for 4024 & 0580 from 2025 to 2027, with great attention and care for every topic. All information is curated in a simple, clear, and concise manner. The aim is to aid students and make learning easier in preparation for their exams. Team Mojza makes every effort to error-check all the content; if you find any discrepancies, please reach out to us at support@mojza.org.